Appendix A

Connection of Magic Chess and Math Puzzles

Frank Ho, M.Sc.

BC certified teacher
Principal of *Vancouver Home School* (**www.bchomeschool.ca**)
Founder of Ho *Math and Chess* Learning Centre (**www.mathandchess.com**)

Email: mathandchess@telus.net
Phone: 604-266-0974

2265 West 41st
Vancouver, BC, Canada
V6M 2A3

April, 2005

Abstract

Magic Chess and Math Puzzles workbook is a new idea of learning chess and also enriching math problem-solving ability using creative minds of hands-on, multi-concept, multi-direction, multi-operation, multi-sensory and visual learning.

Hands-on
Chess provides ample hands-on opportunities. One has to physically move chess pieces to get game going.

Multi-concept
Math puzzles worksheets are designed to learn division while doing multiplication and learning subtraction while doing addition.

Multi-direction
Chess is a multi-direction game. Some worksheets are deigned in a way that the operation is no loner just a linear fashion from left to the right. The direction could be from bottom to top, from left to right, and also diagonally or even crossed.

Multi-sensory
The learning of chess involves eye, hands, and brain coordination. The math pattern puzzle trains visualization.

This paper provides information on why using *Magic Chess and Math Puzzles* would improve a student's math problem-solving ability. The information is provided by extracting smaple problems from the workbook and then gives a corresponding math concept learned by doing the sample problems.
This one- on- one table style comparison gives concrete evidence on what math concepts are learned after working on the math puzzles that are included in the workbook.

Background

I started to teach my son Andrew chess when he was five years old. A lot of information I have read indicates that there is a strong relationship between mathematics and chess. My son was interested in chess so what would happen if he carried the same interest in chess on to a subject which would require him to use his chess knowledge? What could chess offer in mathematics education? These kinds of questions sparked my interest in study the relationship between chess and math. What I had found was there have been many chess or math puzzles published but I was not able to find a collection of mathematical chess problems that was specifically created for youngsters.

In 1995, I was involved in teaching math and chess at the same time and I started to seriously look into the possibility of creating some math and chess hybrid problems. This is how Andrew and I started to create the first generation of Math and Chess integrated workbook entitled *Mathematical Chess Puzzles for Juniors* (ISBN 0-9683967-0-4 and is archived at National Library of Canada). It is designed and written for the purpose of learning all kinds of problem-solving skills through over 100 mind provoking and sometimes mind boggling mathematical chess puzzles. The uniqueness of this book is that only the basic chess knowledge and the elementary math ability are needed to solve most of these puzzles.

I have been working on refining the contents ever since and after 10 years of working on the workbook, it has incorporated many math concepts and today's version is the third generation. The workbook could be used as a supplemental or enriched material to supplement math curriculum.

World's first claim and copyright

I made the claim that the first edition of workbook *Mathematical Chess Puzzles for Juniors* created in 1995 was a world's first. I did a through literature and library catalogues searches and publicized my workbook on the internet (including rec.games.chess.misc) resulted in nothing. There was no integrated chess and math workbook ever published commercially for the elementary students at the time when I published my workbook in 1995. In addition, I have updated the workbook since I published the first one 10 years ago with many new and innovative ideas, no one has ever done or created such a workbook. It would be devastating for me if some copycat just copied my ideas and formats of worksheets and started to commercialize it. To protect my intellectual property, I lay claims as being the world's first on the following ideas and concepts and the original creator of unique formats of worksheets:

- 1995, I incorporated the values of chess pieces into math (The value system is the one used by Chess Federation of Canada.).
- July, 2004, I created two-column format, one column is chess question and the other side is math puzzles, which show the logic relationship between chess and math. This is the second-generation version.
- February, 2005, I used chess figures in the math puzzles to instil the concept of variables. This is the third generation of workbook.

I found that in real life, students do not just learn four operations ($+ - \times \div$) in a sequential manner, for example, when a child taking one apple out of four apples, it is an operation of addition (getting one apple) but also a subtraction (3 apples left). I thought it would be a good idea that students could learn multi-operation and multi-concept at the same time to reflect the real life experience. Many math puzzles worksheets were converted from my math worksheets originated by me.

How math and chess were integrated

The updated workbook today is called *Magic Chess and Math Puzzles* to reflect the magic relationship of mathematics and chess and how chess could be used in improving mathematics ability.
I did not want to create a workbook, which is just a collection of chess puzzles, nor I wanted to create a workbook, which was a collection of brain teasing math puzzles. The problems in my workbook I envisioned to create must have the following characteristics:

1. Must have some logic connection between chess and math problems.
2. Students could learn chess and improve math ability at the same time.
3. Problems created must be innovative and interesting.
4. Only basic chess is required to solve the math problems.
5. Elementary students from grade 1 and above could solve these problems.

With the above requirements in mind I was searching the logic and mathematical connection between math and chess and the result is many of these problems were created using chess positions and are not suitable for elementary students.

In 1995, the breakthrough came to me when I realized that I could use the relative value of chess pieces to create mathematical chess puzzles. I have since created many of these types of problems and I believe this idea of using relative chess piece values to create mathematical chess puzzles is world's first.

In July, 2004, I started to use two-column system to place chess questions on one-side and math problems on the other side to show the possible logic relationship between chess and math. At the same time I was creating math worksheets for elementary students. Some of these math worksheets formats were created by me and they are very unique, I incorporated some of these unique math worksheets into my chess workbook.

In February, 2005, the third generation of *Magic Chess and Math Puzzles* was born. I incorporated chess figures into mathematical equation and the result was a big surprise to me since to some children the learning of using these chess variables in the equations seems to be so natural to them, they are not realizing that they are actually learning algebra at grade 3. At this point I decided to change my workbook name *Chess and Mathematical Chess Puzzles* to *Magic Chess and Math Puzzles* to better reflect the intriguing and magic relationship between chess and math.

Learning expectations

People always wonder what exactly is the relationship between chess and math and how much math concepts in my workbook are children learning? What are the expected learning outcomes? My workbook is not designed to prepare children for the chess tournament, not is it written to prepare for math contest. It is designed to lead children into the wonder world of learning chess and at the same time to introduce some math concepts using chess as a learning tool. It is written with the idea to show children that math is not just doing drill and boring math computations day in and day out. It is offered as an alternative and supplemental learning resources for math education but not to replace school math curriculum. The expected chess-learning outcome stops at the tactics level and could be easily seen from the table of contents of the workbook.

In this article, I would like to give some examples of problems taken right out of my workbook to outline some of the expected learning outcomes of math concepts incorporated in the workbook which also demonstrates on how chess and math could be integrated together to benefit children in improving their mathematics ability.

Chess knowledge required

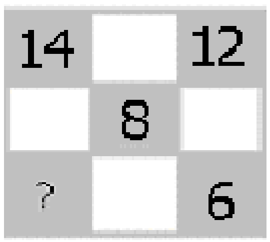
The chess knowledge required to do the mathematical puzzles is listed as follows:

- How to move the chess pieces and how to write moves in algebraic notation.
- The values of chess pieces.
- How to castle.

Addition

Math Puzzle Samples	Expected Math Learning Outcomes	Chess Knowledge Required
Fill in each □. ↓+ ♛ +↓ 1 + 3 ↓ 2 ↓ □ □ □	▪ Adding numbers ▪ Making tens ▪ Substituting unknown variable ▪ Multi-direction operation	▪ Value of chess pieces
♜+ ♞ + x = 45 +♜+ ♞ x = ______	▪ Cancellation ▪ Equation ▪ Substituting unknown variable ▪ Multi-concept learning	▪ Value of chess pieces
Replace each? with a number. 17 ? 9 3 8	▪ Adding numbers ▪ Multi-direction operation	▪ Rook move
♛ + 1 + 8 + 2 + ♛ + 1 + ♛ + 1 + 8 + 2 + ♛ + 1 + 8 + 2 =?	▪ Adding numbers ▪ Making tens ▪ Substituting unknown variable	▪ Value of chess pieces

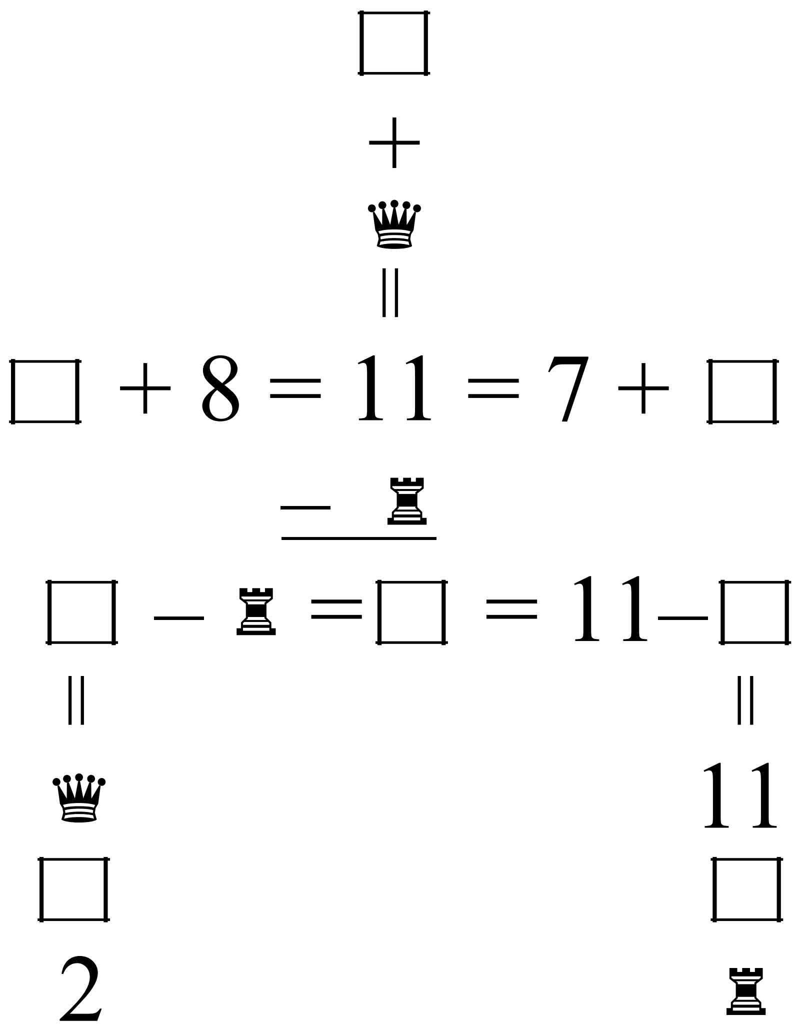
Addition

Math Puzzle Samples	Expected Math Learning Outcomes	Chess Knowledge Required
Replace each? with a number.  	▪ Adding numbers ▪ Multi-direction operation	▪ Bishop move
♜ + ♟ = □ + ♜ + ♚ = □ + ——— ——— □ □	▪ Addition ▪ Comparison ▪ Multi-direction operation ▪ Substituting unknown variable	▪ Value of chess pieces
6 + ♜ (4 + ♟) 6 + ♜ = 6 + 4 + □ = □ = 6 + □ + ♟ └ 10┘ └ 10 ┘	▪ Addition ▪ Comparison ▪ Multi-direction operation ▪ Substituting variable	▪ Value of chess pieces
If ♜ + ♜ = □, then 6 + ♜ must be □. If ♜ + ♜ = □, then ♜ + 6 must be □.	▪ Addition ▪ Comparison ▪ Multi-direction operation Substituting variable	▪ Value of chess pieces

Subtraction

Math Puzzle Samples	Expected Math Learning Outcomes	Chess Knowledge Required
♛ + 8 ――― □ – ♛ = □	▪ Subtracting numbers ▪ Substituting unknown variable ▪ Multi-direction operation ▪ Multi-concept learning	▪ Value of chess pieces
□ □ ――― – ♟ + ♟ ――― 8 8	▪ Subtracting numbers ▪ Substituting unknown variable ▪ Bottom-up operation and then top-down operation ▪ Multi-concept learning	▪ Value of chess pieces

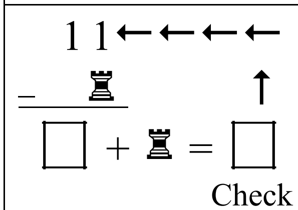
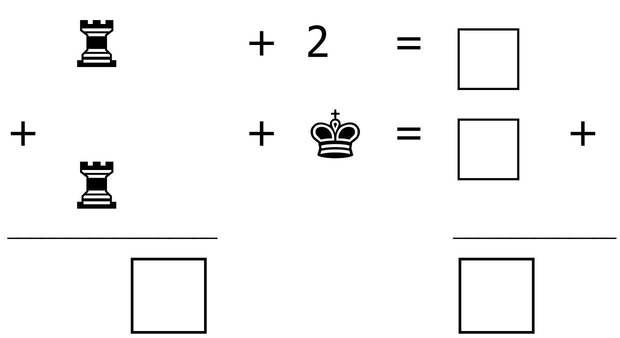
Mixed operations (addition and subtraction)



The above demonstrates muti-direction, multi-concept operations using chess pieces values and variable math concepts.

Expected learning outcomes: addition and subtraction and variable substitution

Subtraction

Math Puzzle Samples	Expected Math Learning Outcomes	Chess Knowledge Required
♜+♞ + x = 63 + ♜ x= _____	▪ Subtracting numbers ▪ Substituting unknown variable ▪ Multi-direction operation ▪ Multi-concept learning	▪ Value of chess pieces
	▪ Subtracting numbers ▪ Substituting unknown variable ▪ Multi-direction operation ▪ Multi-concept learning	▪ Value of chess pieces
	▪ Subtracting numbers ▪ Substituting unknown variable ▪ Multi-direction operation ▪ Multi-concept learning	▪ Value of chess pieces

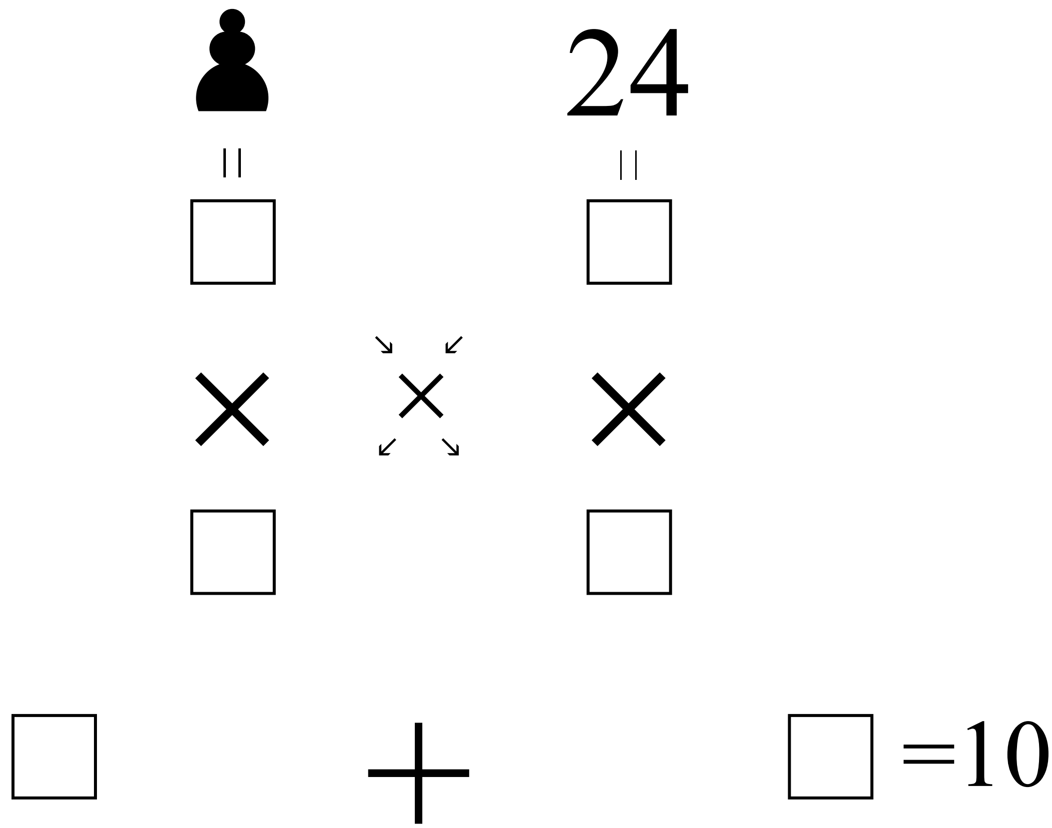
Multiplication

Math Puzzle Samples	Expected Math Learning Outcomes	Chess Knowledge Required
□ □ ↖ ↗ ♜/♜ = 2/2	▪ Cross multiplication ▪ Substituting unknown variable ▪ Multi-direction operation	▪ Value of chess pieces
1 ×6♟ × 7 = □	▪ Multiplication ▪ Substituting unknown variable	▪ Value of chess pieces

Corss Multiplication

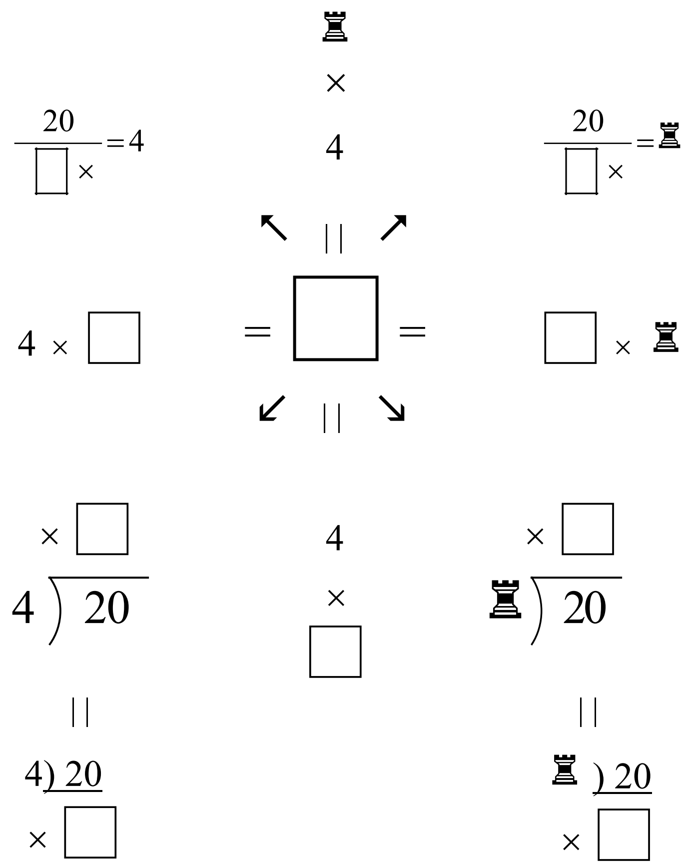
Expected learning outcomes: multiplication, addition, subtraction, Substituting unknown variable

Chess knowledge required: Chess pieces values



Mixed operations

The following is an example of multi-concept learning multi-direction operation.
Expected learning outcomes: multiplication and division



Mixed Operations

The following is an example of multi-concept learning with multi-direction operation.
Expected learning outcomes: addition, subtraction, multiplication and division

	□ – 10	
16 × □	|| = 48 = ||	□ ÷ ♝
× □ 16) 48	6 × □	× □ 4) 48
2) 48 × □	|| □ ÷ ♝ = □ || 8 + □	6) 48

Division

Math Puzzle Samples	Expected Math Learning Outcomes	Chess Knowledge Required
♛ × □ ——— 18 ÷ 2 = □	▪ Multiplication ▪ Division ▪ Substituting unknown variable ▪ Multi-direction operation	▪ Value of chess pieces
(3♛÷3)+(2♜÷5) = 27 ÷3 + 10 ÷5 = 9 + 2 = 11	▪ Multiplication ▪ Division ▪ Substituting unknown variable ▪ Multi-direction operation	▪ Value of chess pieces
♞)180018	▪ Division ▪ Substituting unknown variable	▪ Value of chess pieces

Fraction

One would think that chess perhaps has nothing to do with fractional numbers since all moves are all in whole numbers. Why queen is the most powerful piece in chess and why we usually move chess pieces toward the middle? They all have something to do with the ratio a/b, where b is the 64 squares and a is the squares under control.

Math Puzzle Samples	Expected Math Learning Outcomes	Chess Knowledge Required
$\frac{3♞}{3} + \frac{4♞}{4} = ♞ + ♞ = 3 + 3 = 6$	▪ Fraction ▪ Logic comparison ▪ Substituting unknown variable ▪ Multi-direction operation	▪ Value of chess pieces
$\frac{3♞}{2} = 4\frac{1}{2}$	▪ Convert improper to mixed fraction	▪ Value of chess pieces
$\frac{2}{♜} + \frac{1}{♜} = \frac{2+1}{♜} =$	▪ Adding fractions	▪ Value of chess pieces

Integer

Math Puzzle Samples	Expected Math Learning Outcomes	Chess Knowledge Required
♛ + ♕ = 0	▪ Adding negative number ▪ Substituting unknown variable ▪ Multi-direction operation	▪ Value of chess pieces

Exponent

Math Puzzle Samples	Expected Math Learning Outcomes	Chess Knowledge Required
♛ ♛=9^2	▪ Exponents ▪ Substituting unknown variable ▪ Multi-direction operation	▪ Value of chess pieces

Radical

Math Puzzle Samples	Expected Math Learning Outcomes	Chess Knowledge Required
√♜♜ = ♜= 5	▪ Radicals ▪ Substituting unknown variable ▪ Multi-direction operation	▪ Value of chess pieces

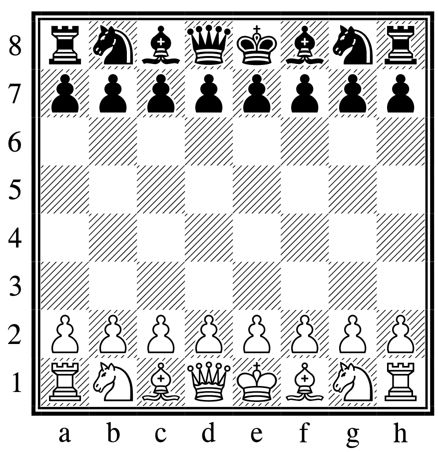
Logic

Math Puzzle Samples	Expected Math Learning Outcomes	Chess Knowledge Required
♜ × 3 ☐ ♜ × 4	▪ Logic comparison ▪ Substituting unknown variable ▪ Multi-direction operation	▪ Value of chess pieces
x+8 - 2 =♛ What is x? _3	▪ Logic comparison ▪ Substituting unknown variable ▪ Multi-direction operation ▪ Equation	▪ Value of chess pieces

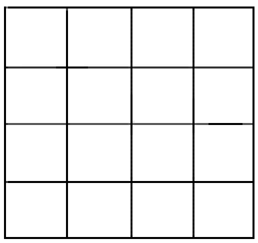
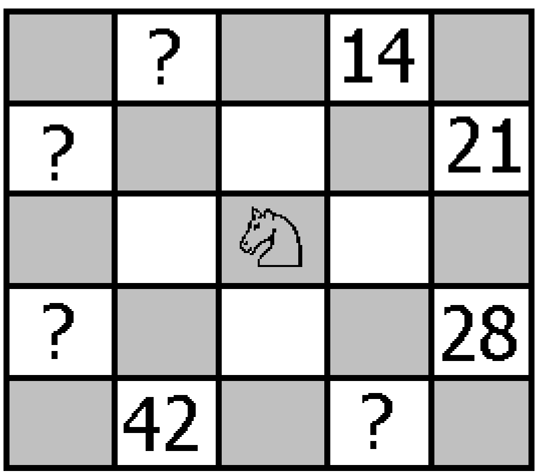
Equation

Math Puzzle Samples	Expected Math Learning Outcomes	Chess Knowledge Required
♜ + x = 63 + ♜ x = _____	▪ Equation ▪ Substituting unknown variable ▪ Multi-direction operation	▪ Value of chess pieces

Visualization

Chess Puzzle Samples	Expected Math Learning Outcomes	Chess Knowledge Required
Write chess notations for the following chess pieces whose positions are in the diagram.  ____________  ___________  ___________ 	If rook is at a1 and is free to make moves along file a and rank 1, what has to be considered before moving? The most important is to see if there are any opponent's pieces, which could intersect with the rook. Thinking in math way would be to see what is y when $x = 1$ and what would be x when $y = 1$, we will be looking for intersections. The idea of coordinates would be easier for chess players to learn if they already have acquired the practical experience of "intersections" coming from different chess pieces. The chess notation is entirely transferable to the concept of math coordinates and vice versa.	▪ Value of chess pieces ▪ Chess piece's move ▪ The ranks and files are related to coordinates.

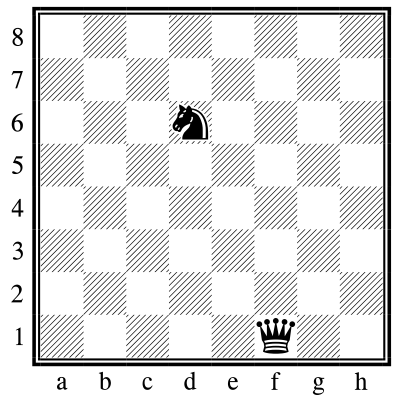
Visualization

Math Puzzle Samples	Expected Math Learning Outcomes	Chess Knowledge Required
How many different sizes of squares are there on the following 4 by 4 square? 	▪ Counting squares	▪ Chessboard
Replace each? with a number. 	▪ Counting ▪ Multiplication ▪ Multi-direction operation	▪ Chess pieces moves

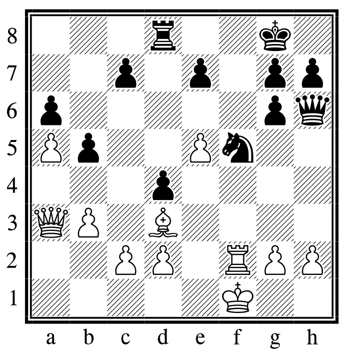
Visualization

<table>
<tr><th>Chess Puzzle Samples</th><th>Expected Math Learning Outcomes</th><th>Chess Knowledge Required</th></tr>
<tr><td>Place the lowest number of white pawns (using letter P) in such a way that the squares numbered are attacked as many times as indicated numbers on the squares.
<table>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table></td><td>▪ Visualization</td><td>▪ Chess pieces moves</td></tr>
</table>

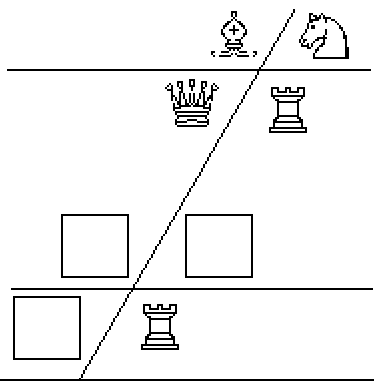
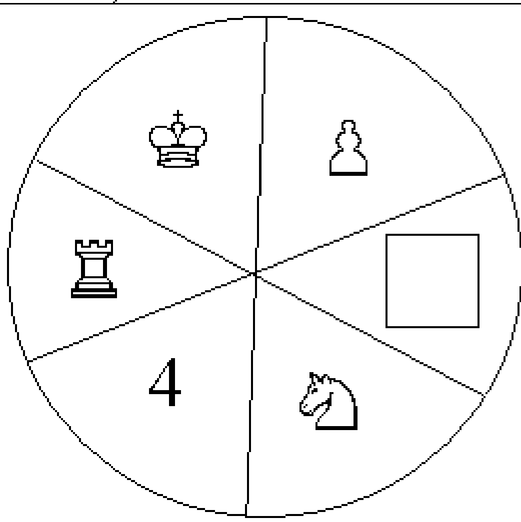
Set

Chess Puzzle Samples	Expected Math Learning Outcomes	Chess Knowledge Required
Cross mark (X) the squares where both chess pieces can move to (intersect). 	Set Find the elements that exist in both sets. ∩ means intersect. {A, B, C} ∩ {A,B}= _____ {1, 2, 3} ∩ {2, 3, 4, 5} = ____ {A, B, C} ∩ {A,B}= _____	▪ Value of chess pieces

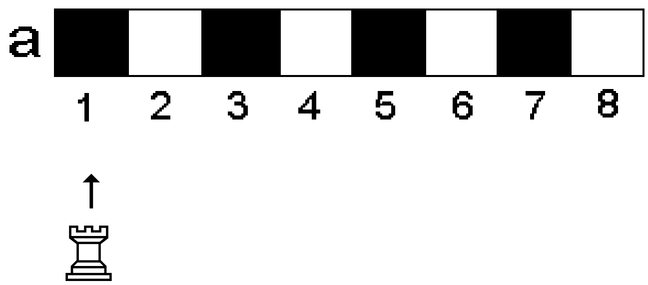
Cancellation

Chess Puzzle Samples	Expected Math Learning Outcomes	Chess Knowledge Required
The way to see which side has more points is not to add up all the total points of chess pieces of each side. Find out which side has more points by cancellation. Cancel pawn with pawn and the same chess piece (or the same number of points) of each side. 	The idea of one-to-one cancellation of chess pieces left on the board is similar to the subtraction property of equation. Evaluate the following. $\frac{1}{2}\times\frac{2}{4}\times\frac{4}{6}\times\frac{6}{8}\times\frac{8}{10}\times\frac{10}{12}$ Do not multiply numbers together first. Cancel numbers whenever you can by having a pair of numerator and denominator divided by the same number.	▪ Value of chess pieces

Pattern

Chess Puzzle Samples	Expected Math Learning Outcomes	Chess Knowledge Required
Observe he following pattern of chess diagram and draw a chess piece in each box. 	▪ Geometry	▪ Value of chess pieces
	▪ Data management	▪ Value of chess pieces

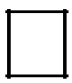
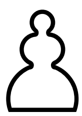
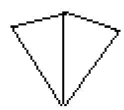
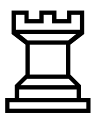
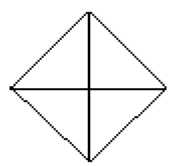
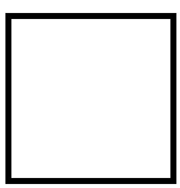
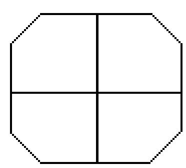
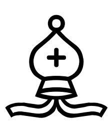
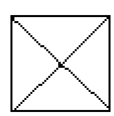
Arrangement

Chess Puzzle Samples	Expected Math Learning Outcomes	Chess knowledge required
In the first rank a, how many different ways can Ra1 move? ________________________ 	▪ Data management	▪ Chess pieces moves

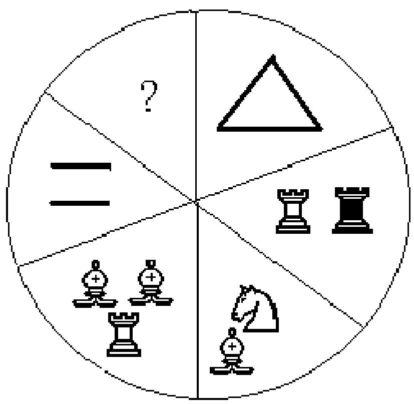
Geometry

The chessboard and chess pieces themselves are geometry. The chessboard is symmetric in terms of its main diagonals. The chessboard is made of 4 identical small boards if it is divided by one horizontal line and one vertical line going through the centre. The set up positions of chess pieces are symmetric between black and white. The chess pieces set up positions on either side is palindrome except the king and queen.

The following uses chess moves to match shapes. Rook's move is a slide motion (left/right, up/down) in geometry. The between moves of rook before reaching the destination is using the concept of communicative property, for example, before Ra1 to Rh1, Rook could move from a1 to c1 (4 squares) then from c1 to h1 (3 squares) or from a1 to d1 (3 squares) then from d1 to h1 (4 squares). The complication is the player has to watch what would happen if the different choices were made and this is much complicated than adding 3 + 4 = 4 + 3 = 7.

Filling in  by a chess piece	Geometric shapes
	
	
	
	

Geometry

Chess Puzzle Samples	Expected Math Learning Outcomes	Chess knowledge required
Find answer to replace the question mark. 	▪ Geometry	▪ Chess pieces moves

If then equation

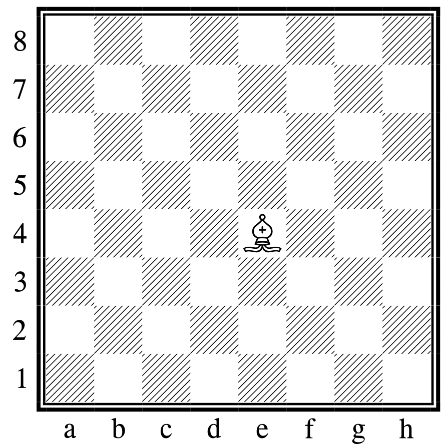
Chess Puzzle Samples	Expected Math Learning Outcomes	Chess knowledge required
Filling in the following □ with a number. If 3 ♖ 2 = 5 then 3 ♗ 2 = □.	▪ Data management	▪ Chess pieces moves

Pattern and relation (Tabulation in $ax + by + ... = c$, where a, b, c are constant.)

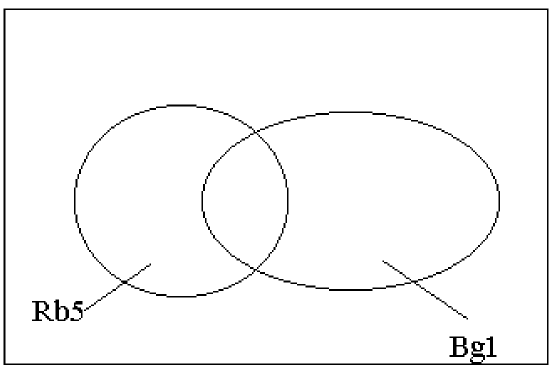
Fill in different number of chess pieces to come up with each total.

Number of ♜	♜ Points	Number of ♞	♞ Points	Total points
1	5	1	3	8 $(1 \times 5 + 1 \times 3 = 8)$
□	5	□	3	11
□	5	□	3	13
□	5	□	3	16

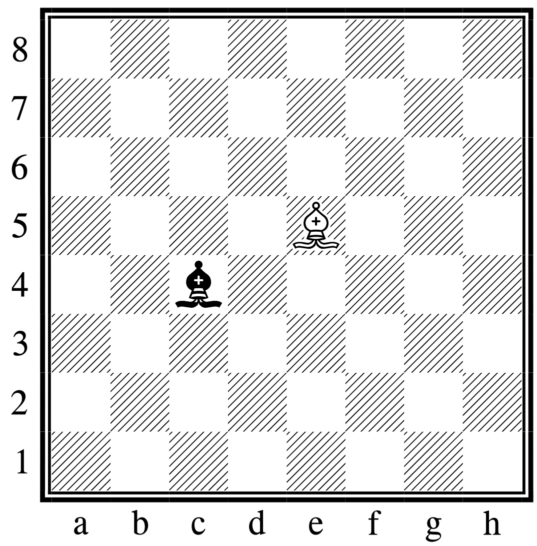
Counting paths

Chess Puzzle Samples	Expected Math Learning Outcomes	Chess knowledge required
How many short paths for ♗ to travel from e4 to f1? 	▪ Data management	▪ Chess pieces moves

Venn diagram

Chess Puzzle Samples	Expected Math Learning Outcomes	Chess knowledge required
The following Venn diagram shows the results of a chess diagram. What squares are only controlled by the rook and the bishop? Answer this question by shading the area on the Venn diagram.  	▪ Venn diagram	▪ Chess pieces moves

Probability

Chess Puzzle Samples	Expected Math Learning Outcomes	Chess knowledge required
What is the probability of the following two pieces meeting together? 	▪ Probability	▪ Chess pieces moves

Tree structure

The calculations of different paths and also the opponent's possible responses are complicated. The deeper the player could calculate the paths, the higher possibility of playing better is. The calculation of path requires logic thinking which is very similar to the idea of using factor tree to find out what are the prime factors of 64, but chess is more complicated in a way, the opponent's moves also have to be thought in advance.

Chess Puzzle Samples	Expected Math Learning Outcomes	Chess knowledge required
Complete the following tree diagram starting at Nb2 (knight moves right and up only.) Nb2 -- c4 -- d6 -- e5 -- d3	▪ Tree structure Find the product of primes of 32. 32 ↙ ↘ 2 16 ↙ ↘ 2 8 ↙ ↘ 2 4 ↙ ↘ 2 2	▪ Chess pieces moves