The Menace Minus Sign

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In elementary and high school, the minus sign (–) is mostly used in mathematics to represent the operation of a subtraction or a negative number but its imbedded dual meaning of both "inverse" and "negative" seems to be largely unmentioned. It could be quite confusing to students. The same symbolism used in different operations, the meaning takes a sharp turn and one of the reasons students do not do well in math is because of this minus sign –.

A minus sign could have different meanings in the following operations:

1. As an operator in subtraction

   A minus is used as an operator, for example, 3 – 2 = 1. In this case 3 – 2 could be explained in a few ways as follows:

   3 – 2 = (+3) – (+2) = 3 – 2 = 1, so we know – (+) = –
   3 – 2 = (+3) + (–2) = 3 – 2 = 1, so we know + (–) = –

2. As a negative sign

   For example,

– 2 + 3 = 1	The minus is an adjective	
3 – (–2)	The first switches the polarity of the second minus. It means the first minus sign is to reverse the sign of – 2.	Example in real life such as $-(-2^0)=2^0$, it means the -2^0 instantly becomes 2^0. We can think it as $y=-(-2)$, so $y=-2$ is flipped over the x-axis to become a positive 2.

3. To change the sign of a value in multiplication

(–2)(–3) = –1 (–3) 2	The first minus sign switches the negative of 3, so the result is positive. We can think of it as (–2)(–3) = –1 $(-3)^1$2, so the first negative sign is to switch the value of $(-3)^1$. It is similar to the idea of $y=-(x-2)^2$, the value of $(x-2)^2$ is flipped along the x axis.

4. Fraction

   What does $-1\frac{2}{3}$ mean?

$$-1\frac{2}{3} = -\left(1\frac{2}{3}\right) = -\left(1+\frac{2}{3}\right) = -1-\frac{2}{3}$$

$$-1\frac{2}{3} \neq 1\frac{-2}{3} \neq 1\frac{2}{-3}$$

$$-1\frac{2}{3} \neq 1+\frac{-2}{3} \neq 1+\frac{2}{-3} \neq 1-\frac{2}{3} \neq -1+\frac{2}{3}$$

The minus in front of the improper fraction means to change the sign of $(1+\frac{2}{3})$ to negative, so it is a good idea to always write negative mixed fraction as a negative improper fraction such as from -$1\frac{2}{3}$ to $-\frac{5}{3}$. Unfortunately the concept of $1\frac{2}{3} = 1+\frac{2}{3}$ does not seem to be explained clearly to the elementary students in math textbooks and students do not understand why the improper fractions need to be converted to improper to do multiplication or negative fraction computations.

Note that $-\frac{2}{3} = \frac{-2}{3} = \frac{2}{-3}$, but how about -$1\frac{2}{3}$? The meaning of negative improper fraction is totally different from negative proper fraction.

5. Take the reciprocal of base in exponent

   For example, 2^{-1} means the reciprocal of 2, which is $\frac{1}{2}$. The exponent -1 has dual meanings, one of them is negative one, but the more important meaning is to take the reciprocal of the base, for example $\left(\frac{2}{3}\right)^{-1} = \frac{3}{2}$. This dual meaning of a minus sign was mostly not explained clearly in the math textbook.

6. Perfect square

   For example, $x^2 = 4, x = \pm 2$. the –2 is hidden and embedded with x.

7. Inequality

   When a minus variable is used in inequality, the inequality sign must be reversed if both side's signs are changed. For example,

   $-x > 1$

   $x < -1$

   The above operation involves 3 simultaneous changes, signs on both sides and the reverse of the inequality, because of this complications, many students got "trapped" in inequality computation.

8. Use as a complementary meaning

For example, to get “at least” probability $1 – p = q$, complementary angle and supplementary angle etc.